Maximal Consistency, Theory of Evidence and Bayesian Conditioning in the Investigative Domain

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1 Introduction

During the upstream phase of an inquiry, much of the detectives’ and magistrates’ everyday task consists of

- acquiring information from investigations on the spot and from witnesses’ depositions
- introducing hypothetical rules to link the various hypotheses into evidential networks
- finding contradictions and incompatibilities inside and across the various depositions
- judging the credibility of the information items
- judging the reliability of the witnesses

In Artificial Intelligence, finding contradictions (or incompatibilities) and rearranging the knowledge base in order to remove them is often referred to as “belief revision”. Somewhat departing from the literature on this subject (see [37, 38] for a survey), in the late eighties Aldo Dragoni conceived a model for belief revision that suited the multi-source characteristic of the inquiry domain [43]. That ATMS-based approach [20] laid down the architecture of an Inquiry Support System (hereafter called ISS) that could help a detective to perform the activities 3, 4 and 5 (ISS was published only five years later [16]). That ISS’s ultimate tasks were those of:

1. finding the maximal consistent subsets of the beliefs about the case under consideration
2. ordering them w.r.t. their global degree of believability.

However, the credibility of the information items, as well as the reliability of the witnesses, were estimated in some strange and rather naive ways. One of ISS’ worst characteristic was that such values were dependent (at least theoretically) on the particular chronological sequence of the depositions received from the various witnesses. Recently the author realized that the “belief-function formalism” adopted to aggregate audit evidence [13, 18, 19] could work as well here, and, especially, that Dempster’s Rule of Combination is a powerful
tool to combine testimonies coming from different independent information sources. By “independent” we mean that each source gives information perceived directly, as an eyewitness, and that s/he does not arrange its deposition with other sources. Furthermore, he also realized that the classic probabilistic technique of Bayesian Conditioning is a good method to estimate the witnesses’ reliability, since it satisfies what he defined “safety norm” (see section 4.1) and since it sets at zero the reliability of a witness that contradicts him/herself (as we believe should be done, see again section 4.1). Consequently, he re-engineered the system to take advantage of the potentiality of these techniques. The resulting new ISS is described in this paper, which is organized as follows. Section 2 introduces the model for belief revision in a multi-source environment that is the core of both, the old and the new ISS. Section 3 presents the Belief-Function Formalism and the Bayesian Conditioning as applied in this multi-sources environment. Section 4 illustrates the new ISS discussing the relevance of these ideas in the inquiry domain. Section 5 contains an example and section 6, written in collaboration with Samuele Animali, discusses merits and limits of ISS. In section 7, Samuele further discusses ISS from a more general perspective, trying to delineate pertinence and confines of any application aimed at supporting the intellectual activity of detectives, magistrates and jurors. The guiding criteria of the planning activity and the obtained results are evaluated taking into account the activity of the user and of the (practical and legal) norms which constraint this activity. This allows us to foresee several characteristics of the ISS, in anticipation of its possible practical exploitation. It is also necessary to note that many of the problems that we have been able to set aside, dedicating ourselves exclusively to the investigative phase, will become inescapable when we challenge more mature phases of the judicial decision making process and problems that require the treatment of juridical argumentation and the knowledge of the legal rules on the part of the system. In effect, a system which, for example, would exploit the deontic logic and be capable of controlling the motivation, in law, of a sentence, would be a model quite different from the ISS.

We conclude this introduction by focusing two “leitmotiv” of this work: the notions of multiple scenarios and comparative evidence. Tillers and Schum showed the importance and the viability of computer-assisted evidence marshaling in the investigative domain (see [44] in this volume), and the works in [45] suggest that juror decision making could be formalized and “computerized” through adequate theories of probability and “cognitive algebra”. However, it seems important to notice that contradictions, which often arise
especially in criminal cases inside a single deposition or among different testimonies, split
the marshaling process into (eventually many) different alternative scenarios, and basic
evidences (i.e. the credibility of the various pieces of information and the reliability of the
sources of information) can be evaluated only on a comparative (not absolute) basis, thus
excluding any “tout court” application of Bayes Theorem (new evidence is quite never
absolute evidence).

2 A Model for Belief Revision in a Multi-Source Environment

Since the seminal, influential and philosophical work of Alchourrón, Gärdenfors and
Makinson [39], the ideas on “belief revision” have been progressively refined [22,37] and
ameliorated toward normative, effective and quasi-computable paradigms [26,29].

Defined as a symbolic model-theoretical problem, belief revision has immediately been
approached both as a qualitative syntactic process and as a numerical mathematical issue.
Trying to give a unitary (although approximate) perspective of this composite subject, we
begin by saying that belief revision can be given both a syntactic and a semantic
characterization, as depicted in Figure 1. The cognitive state \( K \) and the incoming
information \( A \) can be represented either as sets of sentences or as sets of possible worlds
(the models of the sets of sentences). The numbers \( \alpha_i \) can be either reals (normally between
0 and 1), representing explicitly the credibility of the sentences/models, or ordinals,
representing implicitly the believability of the sentences/models w.r.t. the other ones.
Essentially, the belief revision process consists in the redefinition of these weights of
credibility in the light of the incoming information. The resulting revised cognitive state is
generally denoted \( K_A^* \).
Most of the methods for belief revision developed so far obey the following three rationality principles [1,2,22-27]:

BR1. Consistency: $K_A^*$ should be consistent (whatever it means in a numerical setting)

BR2. Minimal Change: $K_A^*$ should be as close as possible to $K$

BR3. Priority of Incoming Information: $K_A^*$ must embody $A$ (this is the reason why $A$ comes without a weight in Figure 1; its weight is taken to be 1)

In a multi-agent environment, where information comes from a variety of human or artificial sources with different degrees of reliability, belief revision has to depart considerably from the original framework. Particularly, the last principle should be abandoned. While giving priority to the incoming information is acceptable when updating the representation of an evolving world, it is not generally justified when revising the representation of a static situation. In this case, the chronological sequence of the informative acts has nothing to do with their credibility or importance. Furthermore, accepting the incoming information (hoping that it is not inconsistent) and remaining consistent, imply throwing away part of the previously held knowledge, but this change should not be irrevocable: for any $B$ derivable from $K$ but not from $K_A^*$, another information $C$ may arrive such that $B$ is again derivable from $K_A^* C^*$ (even if $B$ does not follow from $C$) simply because $C$ causes the rejection of $A$ which previously caused the set-back of $B$, so that now there are no longer reasons for $B$ to remain unbelieved (i.e., $B$ is recovered).
To make practical and useful belief revision in a multi-agent environment, we substitute the priority to the incoming information principle with the following one [28].

BR3. Recoverability: any previously believed information item must belong to the current knowledge space if it is consistent with it.

The rationale for this principle in a multi-source domain is that, if someone gave a piece of information (so that there is someone, or something, that supports it) and currently there is no stronger reason to reject it, then we should accept it!

Along the paper we will represent beliefs as sentences of a propositional language $L$, with the standard connectives $\land$, $\lor$, $\to$ and $\neg$. $\Xi$ is the set of the propositional letters. Beliefs introduced directly by the sources are called assumptions. Those deductively derived from the assumptions are called consequences. Each belief is embodied in an ATMS node [20]:

$<\text{Identifier}, \text{Belief}, \text{Source}, \text{Origin Set}>$

If the node represents a consequence, then Source ($S$) contains only the tag “derived” and Origin Set ($OS$) (we borrowed the name from [21]) contains the identifiers of the assumptions from which it has been derived (and upon which it ultimately depends). If the node represents an assumption, then $S$ contains its source and $OS$ contains the identifier of the node itself. An identical information item may appear in multiple nodes with different $OS$s.\footnote{In particular, the same sentence can appear in an assumption node and in derived nodes at the same time}

We call Knowledge Base ($KB$) the set of the assumptions introduced from the various sources, and we call Knowledge Space ($KS$) the set of all the beliefs (assumptions + consequences).

**Example 2.1.** Let $U$, $W$ and $T$ be three sources, and let $a$, $b$ and $c$ be three atomic propositions. Suppose that $U$ says $b$, $W$ says $a$ and $c$, and finally, $T$ says that $a \to (\neg b)$. Suppose that we (or an assisted theorem prover) deduce $\neg b$ from $a \to (\neg b)$ and $a$. There are five beliefs: four assumptions and one consequence:

$<A_1, b, U, \{A_1\}>$  $<A_2, a, W, \{A_2\}>$  $<A_3, c, W, \{A_3\}>$  $\begin{bmatrix} KB \\ KS \end{bmatrix}

$<A_4, a \to (\neg b), T, \{A_4\}>$  $<C_1, \neg b, \text{derived}, \{A_2, A_4\}>$

\subsection{Maximal consistency and minimal inconsistency}

1 in particular, the same sentence can appear in an assumption node and in derived nodes at the same time
KB and KS grow monotonically since none of their nodes is ever erased from memory. Normally both contain contradictions. A contradiction is a pair of nodes as follows:

\{<_, \alpha, _, >, <_, \neg \alpha, _, >\}

Since propositional languages are decidable, we can find all the contradictions in a finite amount of time. Inspired by Kleer [20], we define “nogood” a minimal inconsistent subset of KB, i.e., a subset of KB that supports a contradiction or an incompatibility and is not a superset of any other nogood. Dually, we define “good” a maximal consistent subset of KB, i.e., a subset of KB that is neither a superset of any nogood nor a subset of any other good.

Each good has a corresponding “context”, which is the subset of KS made of all the nodes whose OS is a subset of that good. Any node can belong to multiple contexts. Managing multiple contexts makes it possible to compare the credibility of different goods as a whole rather than confronting the credibility of single beliefs.

Procedurally, our method of belief revision consists of four steps:

S1. Generating the set NG of all the nogoods and the set G of all the goods in KB

S2. Defining a credibility ordering ≤KB over the assumptions in KB

S3. Extending ≤KB into a credibility ordering ≤G over the goods in G

S4. Selecting the preferred good CG with its corresponding context CC.

Step S1

S1 deals with consistency and works with the symbolic part of the beliefs. Given an inconsistent KB, G and NG are duals: if we remove from KB exactly one element for each nogood in NG, what remains is a good.

Let us recall here the definition of “minimal hitting-set”. If F is a collection of sets, a hitting-set for F is a set \(H \subseteq \bigcup_{S \in F} S\) such that \(H \cap S \neq \emptyset\) for each \(S \in F\). A hitting-set is minimal if none of its proper subsets is a hitting-set for F. It should be clear that G can be found by calculating all the minimal hitting-sets for NG, and keeping the complement of each of them w.r.t. KB.

We adopt the set-covering algorithm described in [14] to find NG and the corresponding G, i.e., to perform S1. This algorithm is briefly described in Appendix A.
Step S2

S2 deals with uncertainty and works with the “weight”, or the “strength”, of the beliefs. A question is: should $\leq_{KB}$ be a symbolic (qualitative, implicit) ordering (relative classification without numerical weights) or should it be a numerical (quantitative, explicit) one? The first approach seems closer to the human cognitive behavior (which normally refrains from numerical calculus). The second approach seems more informative because it takes into account not just relative positions but also the gaps between the degrees of credibility of the various information items. Among the methods of this kind, in our multi-source method for belief revision, we adopted the Dempster-Shafer Theory of Evidence since it provides a very intuitive tool to combine evidences from different sources (see the next section).

Step S3

S3 also deals with uncertainty, but at the goods’ level, i.e. it extends the ordering defined at S2 from the assumptions to the goods. A good is logically equivalent to a sentence of $L$, the conjunction of its elements. Hence, if the method adopted at S2 is able to attach a degree of credibility to any sentence of $L$ (as the Dempster-Shafer Theory of Evidence), then S3 could be superfluous. However, by splitting the two steps one gains in flexibility, being not committed to a single mechanism. In our case, we’ll see that the way the belief-function formalism estimates the credibility of a good is rather unsatisfactory. The method to extend $\leq_{KB}$ into $\leq_{C}$ could take into account either the symbolic ordering or the numerical weights of the assumptions. Let $G'$ and $G''$ be two elements of $G$. Among the former methods, Benferhat et al. [29] suggest the following three ways:

"best-out method. Let $g'$ and $g''$ be the most credible assumptions (according to $\leq_{KB}$), respectively, in $KB\backslash G'$ and $KB\backslash G''$. Then $G''\leq_{C} G'$ iff $g''\leq_{KB} g'$.

"inclusion-based method. Let $G'_i$ and $G''_i$ denote the subsets of, respectively, $G'$ and $G''$, made of all the assumptions with a priority $i$ in $\leq_{KB}$. $G''\leq_{C} G'$ iff a degree $i$ exists such that $G'\supseteq G''$, and for any $j>i$, $G'_j = G''_j$. The goods with the highest priority obtained with this method are the same obtainable with the best-out method.

"lexicographic method. $G''<_{C} G'$ iff a degree $i$ exists such that $|G'|>|G''|$ and for any $j>i$, $|G'_j|=|G''_j|$, and $G''=_{C} G'$ iff for any $j$, $|G'_j|=|G''_j|$.
Although the “best-out” method is easy to implement, it is also very rough since it discriminates the goods by confronting only two assumptions. The lexicographic method could be justified in some particular application domains (e.g. diagnosis). The inclusion-based method seems the most reasonable one since the best goods are obtained by deleting the least credible assumption for each nogood.

**Example 2.2.** Suppose we have the following “stratified” $KB$, with the corresponding goods:

<table>
<thead>
<tr>
<th>$KB$</th>
<th>Good</th>
<th>Superiors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$G_1$</td>
<td>$G_2, G_3$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this case, best-out and lexicographic methods give the same result:

<table>
<thead>
<tr>
<th>Inferiors</th>
<th>Good</th>
<th>Superiors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_2, G_3, G_4$</td>
<td>$G_1$</td>
<td>$G_1, G_2, G_3$</td>
</tr>
<tr>
<td>$G_4$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(if there were a good $G_5=\{(a \land b) \rightarrow c, \neg c\}$ then it would stay at the same level of $G_2$ and $G_3$ with the best-out method, while it would occupy an intermediate position between that level and the level of $G_4$ with the lexicographic one). $G_1$ is the *only* preferred good (since it only has no superiors), while $G_4$ is the *only* least credible one (since it only has no inferiors). $G_2$ and $G_3$ are equally credible. This situation happens when the ordering $\leq_{KB}$ is not strict and the information items that discriminate the goods belong to the same level of credibility. Inclusion-based method yields a somewhat different result

<table>
<thead>
<tr>
<th>Inferiors</th>
<th>Good</th>
<th>Superiors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_2, G_3, G_4$</td>
<td>$G_1$</td>
<td>$G_1, G_2, G_3$</td>
</tr>
<tr>
<td>$G_4$</td>
<td>$G_2$</td>
<td>$G_1$</td>
</tr>
<tr>
<td>$G_4$</td>
<td>$G_3$</td>
<td>$G_1$</td>
</tr>
<tr>
<td>$G_4$</td>
<td></td>
<td>$G_1, G_2, G_3$</td>
</tr>
</tbody>
</table>

In this case, $G_2$ and $G_3$ are incomparable (since $\{a\}$ is neither a superset nor a subset of $\{b\}$). Thus, the resulting ordering is incomplete (but it is strict).
In the special case in which the ordering $\leq_{KB}$ is complete and strict, the three methods produces the same ordering $\leq_G$. In this case, the algorithm in figure 2 (adapted from [34]) is a natural implementation of the method (albeit not the most efficient one).

```
INPUT: Set of goods $G$
OUTPUT: List of goods ordered according to $\leq_G$ in the case that $\leq_{KB}$ is strict and complete
G1 := G
repeat
    stack := KB ordered by $\leq_{KB}$ (most credible on the top)
    G2 := G1
    repeat
        pop an assumption A from stack
        if there exists a good in G2 containing A then delete from G2 the goods not containing A
    until G2 contains only one good G
    put G in reverse_ordered_goods
    delete G from G1
until G1 = ∅
return reverse(reverse_ordered_goods)
```

Fig. 2. An algorithm to sort the maximal consistent subsets (“goods”) of the knowledge base.

Among the “quantitative” explicit methods to perform S3, ordering the goods according to the average credibility of their elements seems reasonable and easy to calculate. A main difference w.r.t. the previous methods is that the preferred good(s) may no longer necessarily contain the most credible piece(s) of information.

**Step S4**

S4 consists of two substeps.

a) Selecting a good $CG$ from $G$. Normally, $CG$ is the good with the highest priority in $\leq_G$.

In case of ties, $CG$ might be either one of those with the same highest priority (randomly selected) or their intersection (see [29] and [30]). This latter case means rejecting all the conflicting but equally credible information items. The result is not a good (it is not maximally consistent) and thus implies rejecting more assumptions than necessary to restore consistency. We believe that this could be avoided by simply considering $\leq_G$ as a primary ordering that could be combined with whatsoever user-defined classification to reduce or eliminate the cases of ties.

b) selecting from $KS$ the derived sentences whose $OS$ is subset of $CG$ to build $CC$. We could relax the definition of $OS$ to that of “the set of assumptions used (but not all necessary) to derive the consequence”. This is easier to compute and does not have pernicious repercussions; the worst that can happen is that, this relaxed $OS$ being a
superset of the real one, is not necessarily a subset of \( CG \) (whenever the real \( OS \) is), and thus the consequence node is erroneously removed from \( CC \).

3 The Belief-Function Formalism in a multi-source environment
We see that the “belief-function formalism”, in the special guise in which Shafer and Srivastava apply it to auditing [13], is a natural way to perform S3 in a multi-source environment. By treating all the evidences as if they had been provided at the same time, this process gives responses that do not depend on their chronological sequence. Its main constraint is that, the underlying probabilistic framework requires the “independence” of the information sources.

The main reference for the Dempster-Shafer Theory of Evidence is still [31]; a general discussion on the belief-function formalism can be found in [32]. The reasons why it fits the requirements of the auditing domain better than the probabilistic approach are explained in [13], while [18] and [19] presents methods to propagate the belief-function values on evidential networks. Here we recapitulate the main concepts, definitions and rules, as they have been exploited in our belief revision mechanism.

3.1 Combining the various evidence sets
To begin with, we introduce two data structures: the reliability set and the information set. Let \( S=\{S_1,\ldots,S_n\} \) be the set of the sources and \( I=\{I_1,\ldots,I_m\} \) be the set of the information items given by these sources. Then:

- **reliability set** = \( \{<S_1, R_1>,\ldots,<S_n, R_n>\} \), where \( R_i \) (a real in [0,1]) is the reliability of \( S_i \), interpreted as the “a priori” probability that \( S_i \) is reliable.
- **information set** = \( \{<I_1, Bel_1>,\ldots,<I_m, Bel_m>\} \), where \( Bel_i \) (a real in [0,1]) is the credibility of \( I_i \).

The reliability set is one of the two inputs of the belief-function formalism (see figure 3). The other one is the set \( \{<S_1, s_1>,\ldots,<S_n, s_n>\} \), where \( s_i \) is the subset of \( I \) made of all the information items given by \( S_i \). The information set is the main output of the belief-function formalism. Figure 3 presents the I/O mechanism applied to the case in the previous example. Let us see now how the mechanism works. Remember that \( \Xi \) denotes the set of the atomic propositions of \( L \). The power set of \( \Xi \), \( \Omega=2^\Xi \), is called frame of discernment. Each element \( \omega \) of \( \Omega \) is a “possible world” or an “interpretation” for \( L \) (the one in which all the propositional letters in \( \omega \) are true and the others are false). Given a set of sentences \( s \subseteq I \)
(i.e., a conjunction of sentences), \([s]\) denotes the interpretations which are a model for all
the sentences in \(s\).

![Diagram](image_url)

Fig. 3. Basic I/O of the belief-function mechanism applied to the previous example.

The main idea with this multi-source version of the belief function framework is that a
reliable source cannot give false information, while an unreliable source can give correct
information; the hypothesis that “\(S_i\) is reliable” is compatible only with \([s_i]\), while the
hypothesis that “\(S_i\) is unreliable” is compatible with the entire set \(\Omega\). Each \(S_i\) gives an
evidence for \(\Omega\) and generates the following basic probability assignment (bpa) \(m_i\) over the
elements \(X\) of \(2^\Omega\):

\[
m_i(X) = \begin{cases} R_i & \text{if } X = [s_i] \\ 1 - R_i & \text{if } X = \Omega \\ 0 & \text{otherwise} \end{cases}
\]

All these bpas will be then combined through the Dempster Rule of Combination:

\[
m(X) = m_1(X) \otimes \ldots \otimes m_n(X) = \frac{\sum_{X_1 \cap \ldots \cap X_n = X} m_1(X_1) \cdot \ldots \cdot m_n(X_n)}{\sum_{X_1 \cap \ldots \cap X_n \neq \emptyset} m_1(X_1) \cdot \ldots \cdot m_n(X_n)}
\]

From the combined bpa \(m\), the credibility of a set of sentences \(s\) is given by:

\[
\text{Bel}(s) = \sum_{X \subseteq [s]} m(X)
\]

**Example 3.1.** Suppose that the case in the example 2.1 was as follows: \(W\) says \(a\), \(U\) says \(b\)

and $T$ says $a \rightarrow \neg b)$. In this case, $\Lambda=\{a, b\}$ so we have four possible worlds: $\omega=\{a, b\}$, $\omega=\{a\}$, $\omega=\{b\}$, $\omega=\{\}$. The correspondences between sentences and possible worlds are as follows: $[a]=\{\omega, \omega\}; [b]=\{\omega, \omega\}; [a \rightarrow \neg b]=\{\omega, \omega, \omega\}$. So we have the following $\text{hpa}$, combined $\text{hpa} m (m_w \otimes m_\Gamma \otimes m_T)$ and relative belief function:

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$m$</th>
<th>$m_\Gamma$</th>
<th>$m_T$</th>
<th>Bel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>$R_w R_\Gamma (1-R_T)$</td>
<td>$R_w R_\Gamma (1-R_T)$</td>
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<tr>
<td>$\omega$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
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<tr>
<td>$\omega$</td>
<td>$(1-R_w) R_\Gamma R_T$</td>
<td>$(1-R_w) R_\Gamma R_T$</td>
<td>$(1-R_w) R_\Gamma R_T$</td>
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<tr>
<td>$\omega$, $\omega$</td>
<td>$R_w$</td>
<td>$R_w (1-R_\Gamma) (1-R_T)$</td>
<td>$R_w (1-R_\Gamma)$</td>
<td></td>
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<tr>
<td>$\omega$, $\omega$</td>
<td>$R_U$</td>
<td>$(1-R_w) R_\Gamma (1-R_T)$</td>
<td>$(1-R_w) R_\Gamma (1-R_T)$</td>
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<td>$\omega$, $\omega$</td>
<td>$R_w R_\Gamma (1-R_T)$</td>
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<td>$\omega$, $\omega$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
<td></td>
</tr>
<tr>
<td>$\omega$, $\omega$, $\omega$</td>
<td>$R_w + R_\Gamma R_w R_\Gamma (1-R_T)$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
<td>$R_w (1-R_\Gamma) R_T$</td>
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</tr>
<tr>
<td>$\omega$, $\omega$, $\omega$</td>
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<tr>
<td>$\omega$, $\omega$, $\omega$</td>
<td>$R_w R_\Gamma (1-R_T)$</td>
<td>$R_w R_\Gamma (1-R_T)$</td>
<td>$R_w R_\Gamma (1-R_T)$</td>
<td></td>
</tr>
<tr>
<td>$\Omega$</td>
<td>$1- R_w$</td>
<td>$1- R_\Gamma$</td>
<td>$1- R_T$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

where all the values in the columns $m$ and $\text{Bel}$ are normalized by $(1- R_w R_\Gamma R_T)$.

Suppose for example that $R_w=R_\Gamma=R_T=0.5$; the output of the formalism is:

$\text{Bel}(a)=\text{Bel}(\{\omega, \omega\})=R_w R_\Gamma (1-R_T) / (1- R_w R_\Gamma R_T) = 0.286$

$\text{Bel}(b)=\text{Bel}(\{\omega, \omega\})=R_w (1-R_\Gamma) R_T / (1- R_w R_\Gamma R_T) = 0.286$

$\text{Bel}(a \rightarrow \neg b)=\text{Bel}(\{\omega, \omega, \omega\})=R_w R_\Gamma (1-R_T) / (1- R_w R_\Gamma R_T) = 0.429$

We can also evaluate the belief value of any set (conjunction) of sentences. In particular we can calculate the credibility of the goods:

$\text{Bel}(\{a, b\})=\text{Bel}(\{a, b\})=\text{Bel}(\{\omega\})=R_w R_\Gamma (1-R_T) / (1- R_w R_\Gamma R_T) = 0.143$

$\text{Bel}(\{a, a \rightarrow \neg b\})=\text{Bel}(\{a, a \rightarrow \neg b\})=\text{Bel}(\{\omega\})=R_w R_\Gamma (1-R_T) / (1- R_w R_\Gamma R_T) = 0.143$

and of the derived sentence:

$\text{Bel}(\neg b)=\text{Bel}(\neg b)=\text{Bel}(\{\omega, \omega\})=R_w (1-R_\Gamma) R_T / (1- R_w R_\Gamma R_T) = 0.143$

As we have seen, the belief-function formalism is able to attach directly a degree of credibility to any good $g$, bypassing step 3 in our framework. The problem is that if a good contains only part of the information items supplied by a source, then its credibility is null! In fact, an assumption is left outside a good $G$ only if it contradicts other assumptions in $G$. Now, if a source $S_A$ has an assumption $A'$ inside $G$ and another one $A''$ outside, then $S_A$
conflicts with some sources supporting assumptions in G. G is supported by a conflicting set of sources, and a conflicting set of sources provides no evidence at all for the information they provide as a whole. It is unreasonable to set at zero the credibility of a good just because its supporting sources are in conflict for pieces of information which are outside of the good; unfortunately, the event is all but infrequent, so that often the credibility of all the goods is null. This is the reason why we adopt the best-out or the “average” method to perform S3 in our system.

3.2 Estimating the “a posteriori” reliability of the sources

In figure 3, we see another output of the mechanism, obtained through Bayesian Conditioning: the set \{<S_i, NR_i>, ..., <S_n, NR_n>\}, where NR_i is the new reliability of S_i. Following Shafer and Srivastava, we defined the “a priori” reliability of a source as the probability that the source is reliable. These degrees of probability are “translated” by the Theory of Evidence into belief-function values on the given pieces of information. However, after all the sources have completed their depositions, we may also want to estimate their “a posteriori” degree of reliability from the cross-examination of their evidences. To be congruent with the “a priori” reliability, the “a posteriori” reliability must be also a probability value, not a belief-function value. This is the reason why we adopt the Bayesian Conditioning instead of the Theory of Evidence to calculate it. Let us see in detail how it works here.

Consider the hypothesis that only the sources belonging to \(\Phi \subseteq S\) are reliable. If the sources are independent, then the probability of this hypothesis is \(R(\Phi) = \prod_{S_i \in \Phi} R_i \cdot \prod_{S_i \notin \Phi} (1 - R_i)\). We could calculate this “combined reliability” for any subset of S. It holds that \(\sum_{\Phi \subseteq S} R(\Phi) = 1\).

Possibly, the sources belonging to a certain \(\Phi\) cannot all be considered reliable because they gave contradictory information, i.e., a set of information items s such that \([s] = \emptyset\). In this case, the combined reliabilities of the remaining subsets of S are subjected to the Bayesian Conditioning so that they sum up again to “1”; i.e., we divide each of them by “1- \(R(\Phi)\)”. In the case where there are more subsets of S, say \(\Phi_1, ..., \Phi_n\), containing sources which cannot all be considered reliable, then \(R(\Phi) = R(\Phi_1) + ... + R(\Phi_n)\). We define the revised reliability \(NR_i\),
of a source $S_i$ as the sum of the conditioned combined reliabilities of the “surviving” subsets of $S$ containing $S_i$.

An important feature of this way to recalculate the sources’ reliability is that if $S_i$ is involved in contradictions, then $NR_i \leq R_i$, otherwise $NR_i = R_i$.

### 3.3 The computational complexity of the Belief-Function Formalism

The main problem with the belief function formalism is the computational complexity of the Dempster’s Rule of Combination; the straight-forward application of the rule is exponential in the frame of discernment (which is $|\Xi|$; however, normally $|\Xi| < |KB|$) and the number of evidence sets. Yet, from an input-output analysis of the mechanism we realized that two properties hold.

1. An information item $I_j$ not involved in contradictions and received from a single source $S_i$, does not contribute to the mechanism, in the sense that the degrees of credibility of the other pieces of information do not depend on it and $\text{Bel}(I_j) = NR_i$.

2. Multiple contradictions involving information items received exclusively from exactly the same sources are redundant; all the pieces of information from the same source receive the same degree of credibility, independently of the number and the cardinality of the contradictions.

Property (1) implies that we can leave temporarily out of the process those sentences received from a single source that are not involved in contradictions. In many cases, this dramatically reduces the size of $\Omega$. Property (2) says that, what is important is that a set of sources was contradictory, not how many times nor about what or about how many pieces of information they did. This also allows us to temporarily leave out of the process some sentences involved in contradictions; this is significant in situations like that of two sources systematically in contradiction only with each other.

Besides these two properties, it is important to say that much effort has been spent in reducing the complexity of Dempster’s Rule of Combination. Such methods range from “efficient implementations” [41] to “qualitative approaches” [42] through “approximate techniques” [40].
4 The Inquiry Support System
The model for belief revision in a multi-source environment described in the previous sections fits very well the investigative domain. This section illustrates the ISS discussing the relevance of its theoretical framework in the inquiry domain.

4.1 The investigative domain
During the upstream phase of an inquiry, much of the detectives’ task consists in
- acquiring information from investigations on the spot and testimonies
- introducing hypothetical rules to link the various hypotheses into evidential networks
- finding the contradictions inside and across the various depositions
- judging the credibility of the information items
- judging the reliability of the witnesses

A testimony is the narration of some facts from a witness experience. The witness reliability has been traditionally considered dependent on his/her accuracy and credibility. Accuracy is the informer’s ability to record and recall from memory the details of the story; it depends on memory, perception and cognition. Credibility depends on the informant’s behavior during the interrogatory (his/her posture) and his/her personal interests in the facts. The witness accuracy can be influenced by many factors. We summarize from various studies [3-11] the following partial list: age, sex, race, elapsed time, various deficits (perceptive, mnemonic or cognitive), stress, stereotypes and prejudices, unconscious memory transfer, interrogatory technique, wish to please.

Regarding the witness credibility, it has been noted that the will to cheat excites the organism, so various semi-unconscious gestures could be considered suspicious: dilation of the pupil, fixing the eyes, winking, smiling, gesticulating, shaking one’s head, continuous movements of legs and feet, delaying the answers, extending the answers, giving irrelevant information, hesitating.

However, in spite of these studies, many researchers think that there is no absolute correlation between the will to lie and any observable physiological behavior.

Also important in judging a witness credibility are his/her personal implication in the case, since a sane person lies only if he has strong reasons to hide the truth. This should imply that a defendant would be less credible than a disinterested external witness who is neither a relative nor a friend or a partner of the defendant himself. However, this is a prick question
because it s obviously dangerous to consider unreliable a person just for his being accused; besides, an unimplicated person could well have unknown reasons to lie.

In conclusion we can say that human intuition seems insufficient to judge a witness reliability, so the spot has recently been moved from the informant’s truthfulness to the information’s credibility. Undeutsch [12] reports various veracity criteria for a testimony; the following is a partial list:

a) amount of details furnished,
b) clearness, precision and vividness of the images presented,
c) narration free from clichés and stereotypes,
d) intrinsic connection of the narration,
e) intrinsic coherence of the narration,
f) coherence of the narration with the physical laws,
g) coherence of the narration with the observed and/or already verified facts.

Evidently, a liar is not oriented to furnish many particulars in his evidence because any detail he offers is an occasion for the magistrate to unmask him. A witness that gives vivid descriptions free from stereotypes is either sincere or is a good dramatic actor. There are some disputes on the fourth criterion. By “connection” we mean that the elements of the story are linked in an acceptable causal/explanatory chain. Often a witness tends to give not just information items but the rules to link them as well. However, many magistrates are not always well disposed toward such testimonies because they prefer to be free to make their own connections in the story and they suspect that the “pre-compiled” links are attempts to divert the inquiry. The fifth criterion seems very important. By “coherence” or “consistency” we mean that the testimony contains no incompatible or inconsistent information items. One of the main tactics of a Public Prosecutor is that of compelling the accused to contradict him/herself; if he succeeds in that, then he is near to winning the suit. It is interesting to notice here that estimating the “a posteriori” reliability of such a witness through Bayesian Conditioning (as ISS does) satisfies this requirement as well since it sets his/her reliability to zero.

The sixth and the seventh criteria may be synthesized into the following one:

g) coherence with the most credible information currently available.

The criteria dealing with coherence (e-g) are captured by Dempster’s Rule of Combination that reinforces concordant evidence and weakens conflicting evidence. In another paper (see the example in [16], p. 965) we said that coherence is more important than connection;
essentially, connection is a rather subjective matter and can be easily artificially constructed to divert the inquiry, while incoherence is a rather objective matter and can hardly be concealed. We can only suspect a witnesses/defendant whose testimony revealed some disconnections, but we can, and do accuse him/her if we find contradictions in his/her deposition.

4.1 Safe criteria to evaluate the reliability of a witness from his/her testimony.

At the time of this, there is a great debate in Italy on the role and the importance of the “pentiti” in the fight against “mafia” and corruption. The Italian phenomenon of the “pentitismo” (confessed culprits turned informers) has an equivalent in, e.g., Britain’s supergrass system. A supergrass is a person who has belonged to an illegal organization and now claims to collaborate with justice. If the conversion is sincere, then the supergrass becomes a very important witness; the problem is that of evaluating his truthfulness and reliability. Mass-media point out the importance of the objective verifications in judging a supergrass’ testimony. Roughly stated, the criteria sound like these:

R1. A supergrass should not be considered reliable until his/her evidence is confirmed by some objective verifications

R2. A supergrass’ reliability should increase with the number and the “importance” of these objective verifications

R3. The more reliable the supergrass, the more credible his/her testimony.

These rules would suggest a very simple criterion to compute mechanically a supergrass’ trustworthiness. Let $s_i$ and $s_j$ be the sets of information items given by $S_i$ and $S_j$. If $o_i \subseteq s_i$ and $o_j \subseteq s_j$ are information items that have been objectively verified, then $S_i$ is more reliable than $S_j$ iff $|o_i| > |o_j|$. The ascertainment of one of the information items in $s_i \setminus o_i$ causes the increase of $S_i$’s reliability and, from R3, the increase of the credibility of all the other still unverified information items in $s_i$.

However, increasing a witness’ reliability after his/her testimony has been partially confirmed by some ascertainment, is very dangerous. A false supergrass can easily stuff his/her testimony with lots of easily verifiable but unessential information items just for the purpose of gaining reliability. If a witness says that:

---

1 if the information items have an associated “degree of importance” w.r.t. the case under consideration, then the comparison should be that of weighted sums of the degrees of importance of the information items.
$I_1 = “The\ sun\ was\ shining”$

$I_2 = “John\ shot\ Ted”$

$I_3 = “(John\ shot\ Ted)\ with\ a\ Colt\ 45”$

Neither the ascertainment of $I_1$ (by some weather-station’s files) nor the verification of $I_3$ (from the examination of the bullet) should increase the credibility of the main core of the testimony, $I_2$; hence the application of the rules R2 and R3 would be misleading.

Instead of increasing a witness’ reliability after his testimony is partially confirmed by the observations, we propose the following criteria:

R*1. Initially, a supergrass should be considered as reliable as the other sources

R*2. A supergrass’ reliability should decrease with the number and the “importance” of the negative objective verifications (safety norm)

By “negative objective verifications” we mean the incompatibility of part of his/her deposition with verified facts. However, a witness’ testimony is prevalently contradicted not by objective verifications but by other witnesses’ testimonies. So we need methods to rearrange the distributions of the degrees of credibility of the information items (1) and the reliability of the sources (2) after the discovery of cross-incompatibilities and contradictions or confirmations among their testimonies. We see that the Dempster’s Rule of Combination provides a simple and reasonable way to perform (1), while Bayesian Conditioning is a right way to perform (2).

4.2 The Inquiry Support System

Figure 4 depicts the basic architecture of ISS, the main blocks of which are explained in what follows.
The inputs
The inputs of ISS are the depositions of the various sources. By “source” we mean every actor which provides information items, i.e., defendants, witnesses, experts and the user of the system, which provides hypothetical facts and rules to link the various information items into evidential networks (U1).

The depositions are represented as sentences of a propositional language (translated from natural language by the user). As long as the case goes on, the atomic propositions are stored for eventual subsequent reuse; it is the task of the user that of avoiding the “open texture” problem by reusing atomic propositions whenever it is possible.

Sentences are immediately converted into clauses, thus the beliefs in $KB$ and $KS$ are clauses.

In the inquiry domain, normally the information items are ground literals (atomic propositions or negation of atomic propositions) so that the conversion has no effect at all.

An interesting characteristic of the inquiry domain is that most of the information items are representable as (sets of) Horn clauses: this assures that contradictions will be detected in linear time!

A consequence of the conversion in clausal form is that while a deposition may be contradictory, a single belief can never be inconsistent.
The Problem Solver
Tasks executed by the Problem Solver are:
PS1. finding the set $NG$ of all the nogoods in $KB$
PS2. finding the minimal subsets of $KB$ that constitute an origin set for a sentence interesting to the user (U2)
We have already defined a contradiction as a pair of nodes as follows:
$\{<_-, \alpha, _, _>, <_-, \neg \alpha, _, _>\}$
In the inquiry domain, dealing with commonsense knowledge, we need to extend this notion to that of “incompatible set of nodes”. An incompatibility is a tuple:
$\{<_-, \alpha, _, _>, <_-, \beta, _, _>, \ldots, <_-, \zeta, _, _>\}$
such that the beliefs $\alpha, \beta, \ldots, \zeta$ cannot all be true; in the user’s mind there is a rule $\chi$ such that from $\{\alpha, \beta, \ldots, \zeta, \chi\}$ a contradiction can be derived. Incompatibilities also could come from software systems for specific tasks as the control of the temporal consistency of the statements [36].

Example 4.1. In the example 2.1, suppose that the atomic propositions $a$, $b$ and $c$ were as follows:
$a$: “the window was open”
$b$: the room was saturated with gas
$c$: the temperature in the room was high
Then $\{A_1, C_1\}$ is a contradiction while $\{A_2, A_3\}$ might be regarded by the user as an incompatibility (coming from a single source!) if he believes in the rule $a \rightarrow (\neg c)$

The Assumption-based Truth Maintenance System (ATMS)
The ATMS maintains the belief dependency structure. Furthermore, it generates the set $G$ of the goods by applying the set covering algorithm (described in appendix A) to the set $NG$ of the nogoods, augmented with all the incompatibilities provided by the user (U3).

The Chooser
The Chooser is comprised of two mechanisms. The Belief-Function process attaches a degree of credibility to each sentence in $K_S$, thus implicitly defining an ordering $\leq_{KB}$ over $KB$. Subsequently, another mechanism extends $\leq_{KB}$ into an ordering $\leq_G$ over the goods. In
U4, the user can choose one (or a combination) of the methods (in our system only the “best-out” and the “average credibility”) described in section 2.

The Belief-Function Formalism is able to attach a degree of credibility to any sentence of $L$. In particular, for each given information item $\alpha$, we can calculate $Bel(\neg\alpha)$, here called “negative belief function values” of $\alpha$. We could take advantage of this capability by modifying the “best-out” and the “average” methods.

" modified best-out method: let $g'$ and $g''$ be the assumptions in $KB$ with the highest negative belief function value, respectively, in $KB\backslash G'$ and $KB\backslash G''$. Then $G'' \leq G'$ iff $Bel(\neg g'') \leq Bel(\neg g')$.

" modified average-based method: the credibility of a good $G$ is the average of the positive belief-function values of the sentences inside $G$ (as in the classic average-based method) and of the negative belief-function values of the sentences outside $G$.

5. Example

The following is a short abstract of a didactic case that ran on the program (written in LPA™ MacPROLOG) which implements the system. The extreme simplicity of the case should ease the comprehension of the basic mechanism. The text in courier is the translation of the original input/output text files from Italian into English. In this example, goods are printed out ordered according to the “best-out” method, however we have provided also their “average credibility” and their belief-function value. The case starts with three sources, $B$, $M$ and $A$, that refer three facts regarding $A$ (which is the defendant in the trial). The sources are assigned the same degree of reliability ($0.5$). The other two special sources are $OBS$ (a fictitious source that give only verified facts; reliability $0.9$) and $U$ (which represents the user and introduces only hypothetical facts and/or rules; reliability $0.3$).

INITIAL INPUT:

$B$ asserts: 'A was a sleeping partner of $R$'
$M$ asserts: 'A was driving the car of $S$'
$A$ asserts: it is false that 'A knew $S$'

"a priori" reliability of $A$, $B$ and $M$: 0.5

There are no contradictions; there is one good and the degrees of reliability don’t change.

| A was a sleeping partner of $R$ | 0.500 |
| A was driving the car of $S$ | 0.500 |
| it is false that A knew $S$ | 0.500 |

GOOD 1

Bel 0.125 Average 0.500
NEW INFORMATION:
U asserts: 'A was using the car of S' implies 'A knew S'

"a priori" reliability of U: 0.3

A was a sleeping partner of R | 0.500
A was driving the car of S | 0.459
it is false that A knew S | 0.459

GOOD 1
Bel 0.189 Average 0.473

GOOD 2
Bel 0.081 Average 0.401

GOOD 3
Bel 0.081 Average 0.401

The hypothetical rule introduced by U causes a contradiction with the testimonies of A and MA. Their reliabilities decrease, but the preferred good does not change (in this case). Note that the second and third goods have a non-empty context (one derived sentence).

NEW INFORMATION:
A asserts: 'L was a friend of A'
OBS asserts: 'L was a cousin of S'

"a priori" reliability of OBS: 0.9

L was a cousin of S | 0.900
A was a sleeping partner of R | 0.500
L was a friend of A | 0.459
A was driving the car of S | 0.459
it is false that A knew S | 0.459

GOOD 1
Bel 0.189 Average 0.556

GOOD 2
Bel 0.081 Average 0.401

GOOD 3
Bel 0.081 Average 0.401
An incoming information that does neither contradict nor confirms anything (as 'A was a sleeping partner of R' and 'L was a friend of A' at this stage) belongs to all the goods; its belief-function value is exactly the “a posteriori” reliability of its source (respectively 0.5 for B and 0.459 for A) as results from the Bayesian Conditioning.

Sources that give information neither contradicted nor confirmed by others (like OBS and B at this stage) maintain their “a priori” reliability. Of course, these general rules simplify enormously the computation.

**NEW INFORMATION:**
U asserts: 'L was a friend of A' and 'L was cousin of S' implies 'A knew S'

<table>
<thead>
<tr>
<th>&quot;a posteriori&quot; rel.</th>
<th>FALL</th>
<th>OLD</th>
<th>NEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBS</td>
<td>0.000</td>
<td>0.900</td>
<td>0.900</td>
</tr>
<tr>
<td>B</td>
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<tr>
<td>A</td>
<td>0.041</td>
<td>0.500</td>
<td>0.459</td>
</tr>
<tr>
<td>M</td>
<td>0.041</td>
<td>0.500</td>
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</tr>
<tr>
<td>U</td>
<td>0.057</td>
<td>0.300</td>
<td>0.243</td>
</tr>
</tbody>
</table>

A posteriori reliability: 0.5 for B and 0.459 for A.
L was a cousin of S | 0.892  
A was a sleeping partner of R | 0.500  
A was driving the car of S | 0.496  
it is false that A knew S | 0.417  
A knew S or |  
it is false that L was a friend of A |  
it is false that L was a cousin of S | 0.184  
it is false that L was a friend of A | deriv

GOOD 3  Bel 0.000  Average 0.498

GOOD 4  Bel 0.000  Average 0.482

GOOD 5  Bel 0.000  Average 0.435

GOOD 6  Bel 0.000  Average 0.403

GOOD 7  Bel 0.009  Average 0.340

"a posteriori" rel. | FALL | OLD | NEW |
B | 0.000 | 0.500 | 0.500 |
M | 0.004 | 0.500 | 0.496 |
OBS | 0.008 | 0.900 | 0.892 |
A | 0.083 | 0.500 | 0.417 |
The new hypothetical rule introduced by \( U \) (the user of the system) yields another contradiction (with 'L was a friend of A' and 'L was a cousin of S'). The reliability of the three sources involved (\( U \), A, and OBS) slightly decrease. Also the credibility of all the information items they provided decrease, not only the credibility of the propositions directly involved in the contradiction (see the information 'A was using the car of S' implies 'A knew S' that falls from 0.2432432432 to 0.1836734694).

Until now, the hypothetical rules introduced by the user have been discarded from the preferred good. The hypothesis 'A knew S' belongs to the context (derived sentences) of the third good. Suppose now that it comes out a letter of recommendation written by A to S. This can be taken as a proof that 'A knew S'.

**NEW INFORMATION:**

OBS asserts: 'A knew S'

\[
\begin{align*}
\text{A knew S or} & \quad \text{or} \quad \text{it is false that L was a friend of A} \quad \text{or} \quad \text{it is false that L was a cousin of S} \\
\text{A knew S} & \quad \text{or} \quad \text{it is false that A was driving the car of S} \\
\text{A knew S} & \quad \text{or} \quad \text{it is false that A was driving the car of S} \\
\text{A was a sleeping partner of R} & \quad \text{or} \quad \text{it is false that A knew S} \\
\text{L was a friend of A} & \quad \text{or} \quad \text{it is false that A was driving the car of S} \\
\end{align*}
\]

**GOOD 1**

\[
\begin{align*}
\text{Bel} & \quad 0.000 \\
\text{Average} & \quad 0.641 \\
\end{align*}
\]

**GOOD 2**

\[
\begin{align*}
\text{Bel} & \quad 0.000 \\
\text{Average} & \quad 0.630 \\
\end{align*}
\]

**GOOD 3**

\[
\begin{align*}
\text{Bel} & \quad 0.014 \\
\text{Average} & \quad 0.480 \\
\end{align*}
\]
L was a cousin of S | 0.829
A was a sleeping partner of R | 0.500
A was driving the car of S | 0.493
L was a friend of A | 0.078
it is false that A knew S | 0.078

GOOD 4 Bel 0.000 Average 0.396

A knew S or |
it is false that L was a friend of A or |
it is false that L was a cousin of S | 0.871
L was a cousin of S | 0.829
A was a sleeping partner of R | 0.500
A was driving the car of S | 0.493
it is false that A knew S | 0.078
it is false that L was a friend of A | deriv

GOOD 5 Bel 0.000 Average 0.554

A knew S or |
it is false that L was a friend of A or |
it is false that L was a cousin of S | 0.871
L was a cousin of S | 0.829
A was a sleeping partner of R | 0.500
A was driving the car of S | 0.493
L was a friend of A | 0.078
it is false that A knew S | 0.078
it is false that L was a cousin of S | deriv

GOOD 6 Bel 0.000 Average 0.404

A knew S or |
it is false that A was driving the car of S | 0.871
L was a cousin of S | 0.829
A was a sleeping partner of R | 0.500
L was a friend of A | 0.078
it is false that A knew S | 0.078
it is false that A was driving the car of S | deriv

GOOD 7 Bel 0.000 Average 0.471

"a posteriori" rel. | FALL | OLD | NEW |
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<thead>
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<tbody>
<tr>
<td>B</td>
<td>0.000</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>M</td>
<td>0.007</td>
<td>0.500</td>
<td>0.493</td>
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<tr>
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<td>0.300</td>
<td>0.290</td>
</tr>
<tr>
<td>OBS</td>
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<td>0.900</td>
<td>0.829</td>
</tr>
<tr>
<td>A</td>
<td>0.422</td>
<td>0.500</td>
<td>0.078</td>
</tr>
</tbody>
</table>

The previously preferred good now is the least preferred! The hypothetical rules introduced by the user are now far more credible than how much reliable is the user himself. A is now the least reliable information provider.

During the working session the user can query the system about various items, for instance:

- which and how many are the information given by a certain witness
- which assumptions are involved in at least one nogood
- find the more credible context that contains a certain assumption
6. Discussion about ISS
The main improvements w.r.t. the system presented in [16] is that the credibility of the information items are revised through the Dempster-Shafer Theory of Evidence. Each source of information is regarded as an evidence about the facts of the story. Each source has a weight, its “a priori” reliability, and all these weighted evidences will be combined through Dempster’s Rule of Combination.
An advantage of this approach is that all the information items are treated as they had been received at the same time, so as to avoid the problem of “chronological dependency” (different sequence of information items yields different results). These changes render the system more intuitive to most readers, without impacting on the following limits and merits.

6.1 Criticisms
- Criticism. ISS attaches a single global measurement of reliability to a given source while that value should depend on the various information items coming from that source. The same source could be considered reliable when s/he says $\alpha$ and unreliable when s/he says $\beta$, or vice-versa, hence its degree of reliability should be a function whose domain is the set of pieces of information given by that source. What ISS really measures is a sort of general attitude of the source to give false information (intentionally or not), independently from the particular domain of the discourse. Sometimes that measurement could be too coarse.

Answer. An obvious solution could be that of partitioning a witness’ (e.g., John_Doe) deposition as it were given by different persons (i.e., John_Doe-professional_man, John_Doe-father, John_Doe-politician ...) with different degrees of reliability. However, this method has two serious drawbacks. First, it introduces two levels of arbitrariness, in the definition of the different functional charachterizations of the witness, and in the definition of their corresponding degrees of “a priori” reliability. Second, by multiplying the sources, it increases exponentially the computational cost of the Dempster’s Rule of Combination. However, this “schizophrenic” approach has an interesting consequence: a witness under a characterization A may contradict
him/herself under a characterization B without losing completely his/her reliability (the conflict is treated as if it would happened amongst two different persons).

More generally, a criticism of this type indicates the possibility of a misunderstanding. It is undeniable that the detective or the magistrate who is conducting the investigation may face with individuals who lie at times, and who, at times are sincere, but this does not depend on the domain. Besides, the reliability of the source, on the whole, is undermined even by one single untruth. Every defense attorney tries to draw the witness for the prosecution into contradiction, because the source is discredited by inconsistency. This is true even if the contradiction regards details which are not relevant in a logical and coherent reconstruction of the facts. A person aware of the facts who gives the police an inaccurate or incorrect information automatically puts him/herself in a bad light, regardless of the relevance of that detail. Every detail s/he gives will be considered with circumspection and prudence. This does not mean that the source can no longer be considered as reliable. S/he may still furnish useful information, but the pieces of information given are not seen as totally independent pieces of knowledge; instead, their significance is tied to their source, to the circumstances in which they are given, and so on. In particular, the statements given by people who are informed as to the facts become fully significant only when seen in the light of the context in which they are given. In fact, they are not separate pieces of information, but a series of statements given in specific conditions and taken within the framework of certain rules that make them suitable as evidence (to this end the drafting of a written statement, the presence of an attorney, the following of correct procedures on the part of the investigating authorities, etc. are important). It is true that the investigator can also use qualified information, based on the particular expertise of the person who gives the information. These are experts who put their scientific or technical knowledge to the disposition of the investigating authorities, applying this knowledge to the case at hand. But the witness is never considered as an expert, even if by chance s/he possesses scientific, technical or specialized knowledge which can be utilized in the specific case under investigation. This knowledge allows him/her to give a more detailed description of the events about which s/he is called to testify in the investigation, but his/her general interpretations, based on his/her human or professional experience, does not constraint the investigating authorities’ ones. We could say that the witness is the expert of that specific case. This and only this is
important to the investigation and to evaluate his/her reliability. The witness’ specific (technical) ability may allow him/her to see, hear or interpretate better what might escape a non-expert witness. An expert witness is more protected from the possibility of error, but not from the possibility of lying. However, it does not influence reliability, given that the source is called to refer what s/he has seen or heard, while the evaluation of the information given is the responsibility of the judge, or perhaps of an uninterested expert, in the case where technical knowledge is to be used. In conclusion, the criticism that the degree of reliability of a given source should depend on the particular information item coming from that source do not necessarily have a counterpart in practice.

• **Criticism.** In estimating the reliability of a witness, ISS disregards the motivations, his/her intentions and personal implications in the case under consideration. ISS does not distinguish insincerity from incompetence; as a consequence, it doesn’t discriminate in any way a witness from a defendant.

  *Answer.* This should not necessarily be considered a limit, but a form of warranty for the defendant. However, in countries where any accused who lies does not commit a crime (as in Italy), it seems reasonable to some people (but not to us) to attach a lower degree of “a priori” reliability to the defendant.

More generally, once again, it is a matter of not trying to give precedence, at all costs, to what may be true in theory with respect to the consideration of the social process which ISS is part of. The automatic system is not the alter-ego of the user; it is the user who, by furnishing the relevant pieces of information to the system, makes the most significant selection. This selection is fruit of the choice made by the user. The person conducting the investigation has distinct freedom in considering the motivations and the involvement of the witness, even in those cases where regulatory precepts furnish the investigator with an orientation. These choices affect the results of ISS. During the investigative stage, the distinction witness-defendant is considered a work hypothesis and a source of particular guarantees for the suspect, while it only assumes its full significance subsequently, in the trial stage. Therefore, a rule which generally requires the system to take this distinction into account, would not be justified. It is the user of ISS who will evaluate the opportunity to give more or less significance to this distinction. To specify a presumption of guilt would only cast
prejudice on the investigative proceedings. Here come into play the distinction between very different models of Justice and of State [46].

- Criticism. In estimating the reliability of a witness, ISS takes into account only what s/he said, not what s/he didn’t say while it should have been said.

Answer. True. For the moment, ISS doesn’t deal with the incompleteness of the depositions and it doesn’t distinguish between spontaneous and extorted depositions.

It is not clear to me how to estimate the reliability of a witness whose deposition has been extorted.

We can, however, make several observations. It is true that reticence is negatively evaluated during the investigation. But it surely has not been shown how reticence affects reliability, much less if it affects it in an univocal direction. Therefore, it is not appropriate to take it into consideration at this level.

In the investigative stage, that which has not been said does not necessarily affect the truthfulness or the falseness of the statements given. It is true that reticence may lead to suspicion (for example that the source wants to protect someone). The same reticence, however, may simply be fruit of a scrupulous interpretation of the role of the informant: the person called upon to give information to the investigators is not sure of what he/she has seen or heard and, therefore, avoids making incriminating statements. The association between unreliability and reticence would, therefore, be the result of prejudice. The inappropriateness of automatically indexing this prejudice in the system is evident. Furthermore, the user's prejudice reflects on the results elaborated by the system, in as much as it affects the information input into the system. In effect, ISS is not able to correct this type of prejudice, but only carry it to its final consequences.

A different argument could be made in discussing the evaluation of the evidence. After the fact, at the moment of the evaluation, the judge or the juror may certainly take into account reticence in forming his/her own opinion. But this type of problem, at least for now, lies outside of the confines of the field of application which we are delineating for the ISS.

6.2 Merits

- The behavior of ISS can be interactively influenced and controlled by the user in four ways. Referring to the figure 3, the user can:

U1. freely introduce hypotheses suggested by his/her human and professional experience
U2. freely introduce derived sentences
U3. freely introduce incompatibilities amongst the assumptions
U4. freely change the reliability values of any sources; in this way the user can recover all those abilities to judge which come from his/her human and professional experience
U5. freely change the preferred context selecting his own preferred one, for instance the first one containing a certain assumption which he considers particularly credible, and so on.

• The way ISS recalculates the reliability of the various witnesses (through Bayesian conditioning) respects the safety norm: “do not increase the reliability of a witness whose deposition has been confirmed, but decrease the reliability of a witness whose testimony has been disconfirmed”.

• ISS has been considered useful both by detectives and magistrates. The former point out its utility during the preliminary inquiries, the latter suggest its employment after the trial’s conclusion; if the sentencing is based on motivations and judgments incompatible with the information in the preferred context furnished by the ISS, then the judges may well have to explain their decision.

• During the inquiry (or the trial) the inspector (or the magistrate) must carefully avoid prejudice. In particular it would be better if the investigation's bias is not conditioned on the chronology of the informative events. This means that the current detective's opinion regarding the credibilities of the evidences and the reliability of the witnesses should be the same as all the information about the case would have been received simultaneously. Such a machinery could support the user in this important ability.

6.3 Future work and developments

• We would like to represent the structure of the proofs graphically. The most significant information structures in the system are the good and its associated context. Each context can be associated with an and/or graph whose nodes are its atomic information items, while the arcs represent its rules (those given by the various information sources) and the hypothetical rules of the Reasoner eventually involved in the production of an “autohypothesis” inside the context. We could define a module that automatically builds and represents graphically the structure of each context; in
this way one could compare not just flat sets of atomic pieces of information but alternative proof structures.

- A major conceptual improvement would be that of automatic extraction of advice by the system, regarding the future course of the inquiry. We could apply some techniques (like the “minimum entropy” one, see [35]) to drive the acquisition of new evidence regarding the case under consideration, in order to further differentiate among the goods.

7. General Analysis and Perspectives

The ever growing diffusion of electronic data processing clearly affects work methods and techniques, the structure and the operation of organizations and power structures. Little by little, it has assumed the connotations of a real catalyst of social and cultural change. This has also taken place in the juridical field. The computerized archiving of data permits a significant increase in the speed of finding information, of consulting data, of processing the same; information support systems can enhance the visibility and transparency of the apparatus responsible for social control and justice administration, and so on. But, it still has not arrived at the point of “applying law” through the use of automatic systems. In that case, in fact, a whole series of problems arises and, in order to resolve them, relative literature re-examines some of the questions most frequently and copiously discussed regarding the theory and philosophy of Law. These questions regard the relationship between logic and Law and between natural sciences and juridical sciences, the specificity of juridical reasoning or the possibility of formalizing Law from first principles only.

A system like ISS appears on the threshold which introduces the application of artificial intelligence to the field of Law. This confirms the ambition that an automatic system model should be able to be efficiently used in practice.

The success of electronic data processing tools, and of the techniques that derive from this success, seems to be mostly ascribable to the fact that they bring about a perceptible increase in the efficiency of the sociotechnical systems in which they are used. But this depends on the functions that the processing system is able to perform once inserted in a specific social process. Therefore, it is necessary to define the relationship and the reciprocal conditioning going on between the technological tools and the application contexts in which they operate. The tool is modeled on the work for which it will be utilized, but its use will change the very applications which inspired its creation [47,48,57]. The dialogue between the systems engineer and the jurist allows the first to grasp the features from the field of application and the bounds which s/he must consider in creating
models acceptable to the disciplinary contexts of application. These features and bounds are translated into requirements that the model must satisfy. The results of the investigation flow together into the institutionalized decisional process which is the judicial procedure. It strongly influences the development of the investigation. Even if it is true that it is not yet exactly judicial decisional activity, it does nonetheless require the person making the evaluations to operate within a context strongly regulated by the Law. It could not be otherwise, given that the destination of the product of this activity is the judge. But this is not sufficient.

The problems connected to the architecture of an Inquiry Support System are also evaluated with the regulatory prospects in mind and with the sociocultural implications that they represent. Only in this way we can attribute significance to characteristics and requisites that, from a theoretic point of view, could be also seen as system defects. In evaluating the characteristics of ISS, we directly consider the criteria that guide the actions of the user who employs the technological tool. Therefore, not only the criteria assigned to that user, but criteria that he/she really observes. Sociological research shows that the latter does not coincide with or, in any event, does not find its end in the former.

The work of the investigative authority is influenced by prejudices or, perhaps, by the career ambitions of the person who concretely carries out or directs the investigation, by the pressure of public opinion, by informal praxis, such as the promises of impunity in exchange for information.

The ISS user, whether detective or magistrate, if on the one hand is held to the observance of specific regulations, on the other, focuses on and interprets the regulations, and more generally, acts in the light of his/her personal and technical experience. Thus, s/he utilizes the know-how that s/he has gained from experience and puts him/herself in relationship with models which derive from his/her belonging to a specific professional ambit, with consolidated practices, with written or unwritten laws to be respected or, at any rate, which require that s/he take a precise position [59].

Various disciplines, and in first place the juridical disciplines, tell us much about how a judicial investigation should be conducted. But what also interests us is what the operator -the system user- really does and thinks. And the user in question employs the prescriptions that are given to him/her as a canvas with which s/he, more or less openly, relates to in a dialectic manner. The investigator makes use of his own personal science, using secret informers, or perhaps, tools even more impudent, such as unauthorized environmental
interceptions or vaguely disguised intimidation. It is highly unlikely that all of this can find space within automatic models. This does not mean that these models do not, or cannot, say anything to the investigator; it simply leads us to define carefully the confines within which the system can integrate the work of the investigator, or can furnish him/her a service that s/he is able to appreciate.

In the ISS, the approach based on “belief revision” avoids some of the main problems that models destined to be used in a juridical context must inevitably deal with. In particular, the orientation towards the value of knowledge that characterizes scientific investigation finds its limit in other "values" of the juridical context.

The knowledge that originates from the investigation and trial cannot be utilized if it has not be acquired according to specific rules and procedures. The rules and the procedures must be first respected during the investigation, and then during the trial; nevertheless, unlike the methodological rules that preside over the scientific investigation, the rules and procedures in question not only must ensure inter-subjective controllability, but are dictated by an ulterior collective interest. The need to safeguard personal dignity and liberty is, in fact, translated into a series of guarantees relative to the dignity and the civility of the tools used in crime repression. This influences the methods of forming and acquiring the evidence. The historical facts become relevant in a juridical sense reconstructed according to criteria typical of the juridical field [49]. As we have stated, these questions fall outside of the sphere of ISS.

ISS identifies logical contradictions that must be considered as such, even by those who must deal with juridical rules. The results of the operations completed by the ISS suggest evaluations, but do not produce new hypothetical information that has not already been prefigured in the data that the system processes. It can prevent the operator from committing certain types of banal errors. But every juridical evaluation is already inherent in the input, and the system is limited to developing it. Therefore, in addition to being logically correct, the result of the process can also be appreciated by the legal system because it protects the subsequent decision from several logical defects and leaves the prerogatives left to the juridical operator unaltered.

The “serving” role of technology in the juridical field becomes evident in this phase of automatic system development. Perhaps it has to do with a weak approach to artificial intelligence and to the expert juridical systems. It is, however, an approach that we retain to be particularly practical and concrete.
C. Perelman observes:

“If one remains exclusively at the level of the act, the perfect judge would be similar to a machine able to give the answer when he knows the elements of the problem, without worrying about what is at stake and who would be the victim of a possible error”

given this:

“It would possible to realize the ideal of juridical positivism in a juridical system sufficiently well developed, clear, and complete, so that as to be managed by an automaton.” (in [50], pp. 110-11, our translation)

But even if it is true that the statue of Justice is blindfolded, nevertheless, the idea of likening the law to a technique where given a just and correct procedure, the result is just and correct, whatever it may be, as long as the procedure is followed, is not confirmed in the theory and practice of Law. Furthermore, the same presumable impartiality “of the system” is without doubt conditioned a priori by the partiality of the evaluations made by the person who enters the data to be processed into the system, and can be conditioned by the more or less explicit partiality of the choices made by the system designers, who leave their mark in the architecture of the system.

The performance of the ISS consists in making available to the operator rules that have been fine tuned through numerical processing instead of less fine tuned rules belonging to qualitative judgment; rules that have a tradition of applicability (ex. The theory of probability), rather than subjective rules.

The ISS presents itself as a system efficient in highlighting the incongruity present in a cognitive state, and incapable of substituting the user, whether the user be a detective or a magistrate. Only the user is master of the game. This strong dependence on the input of the processed results permits the operator to control the system and, in the extreme case, to knowingly manipulate the system in order to lend support to his/her own convictions. In the case of the ISS, the use of the automatic system, therefore, does not justify (nor does it cause) any type of absolution of the human operator’s responsibility with respect to the decision. Instead, a more solid foundation is given to the decision, increasing its resistance with respect to possible appeals. The decision can be said to be more solid, in that the system is able to ensure the absence of contradictions in the ambit of the cognitive state in which the decisions are framed. Fewer contradictions mean fewer points of friction between the prosecutor and the defense during the justice procedure, and fewer motives for
appealing an indefinite sentence, even if we certainly do not yet have the exact perception of how much the use of a system like the ISS may be an influencing factor from this point of view.

The operational limits of the ISS are, in the final analysis, its strong points. The idea of constructing system models intended to directly influence the decisional phase of the trial highlights a series of problems that we merely indicate here, without the pretext of being analytical or exhaustive, but only of outlining the possible developments of the described model.

If we leave the straits of a merely declarative concept of judiciary activity (the judge like “bouche de la loi” [51]), we can easily see how political options in the activity of administrating justice are manifest at many levels and on multiple occasions. The existence in society of interests, ideologies and projects that are in line with those of state authorities, invests magistrates with ulterior roles in addition to those formally attributed to them. And so, the magistrate, from time to time, assumes the role of legislator, mediator, administrator, comptroller, social worker; the juridical tools also reveal themselves to be tools for satisfying social and political demands.

The idea of supplying the legal universe with procedures that, to the greatest extent possible, are objective and impersonal (not to speak of the possibility of bringing the juridical phenomenology to a generally accepted axiomatic regulation), comes up against difficulties that are unlikely to be surmounted. In particular, the temptation of objectifying, into a syntactic model, the knowledge managed by the juridical system, risks abstracting that knowledge from the wider context of juridical experience, thus underestimating the undeniable presence of evaluational and dialectical moments in juridical argumentation.

The same technique of sanctions, that is also object of a regulatory system, in general highly analytical, offers a wide variety of options, that are resolved with ample discretion. Making reference to the Italian national normative, we can point to the commensuration of the penalty, the judgment pronounced on circumstances which aggravate or attenuate the crime, but also to the evaluation concerning the subsistence of the suppositions for applying provisory protective measurements and of the verification of the subsistence of suppositions for the admissibility of differentiated rites.

The flexibility and the possibility of deviating from the praxis to several procedural formalisms serve to ensure a greater substantial justice, there where the abstractly
equalizing structure of justice allows the advantages of power, wealth, and experience to reproduce in the trial in favor of one part over the other ([52], p. 158).

The judge, but also the magistrates to whom the prosecution is entrusted, and the police force, exercise a form of control over the administration, they develop a real vision of the interests and needs of society and their determination, they enter in conflict with the policies of the other principal decision makers, sometimes to the point of assuming a re-equalizing attitude with respect to antagonizing social relationships. In this way, they are interpreters of social conflict, first, and then of the law; they carry out an authentic work of articulating social claims. Italy has seen many different manifestations of that which has been called “substitute” justice, for example in the field of environmental politics. The phenomenon has assumed surprising proportions with the “Mani Pulite” (clean hands) operation, in which a restricted group of public attorneys, assisted by the police force, created the conditions for ousting the most highly visible part of an entire political class [53,54, 56].

Although none of them finds verification in the operative possibilities of the expert systems, the features of ISS suggest a moderately optimistic vision of the future of expert systems in the juridical field (although, we repeat, the ISS does not process juridical knowledge). Considering the characteristics, and the properties, of the expert systems the use of an automatic tool in the juridical field, which is controllable and which has strictly defined functions, is plausible. On the basis of a logical model of the juridical normative and on the basis of legally important facts, through logically correct inference, a suitably programmed electronic processor could highlight hypotheses to be submitted to ulterior examination by the juridical operator. Therefore, the lines of reciprocal compensation remain delineated between a system based on knowledge, and human decision. There probably exists a point of equilibrium between the power of the models that avail themselves of automatic calculation and the capacity of overcoming the impasses that, at present, belongs exclusively to the human operator [58].

In particular, a possible field of use for the approach which characterizes ISS is the control of decision motivation. ISS lends itself to verifying the coherence of the reconstruction of the facts as laid out in the motivation, as well as the argumentative coherence of the justification of the decision (it is evident in the second case that the system will have to directly challenge the juridical rules, with the problems that we have tried to highlight, above). A logical defect on the motivation, at least in the Italian system, constitutes a motive for appealing decisionary measures. It is not necessary to specify that the system
lends itself to verifying the logical coherence of motivation, even with regard to decisions eventually reached in a completely autonomous manner with respect to what the system may have “suggested”. Even if the motivation logically and materially precedes the disposition in the final redaction of a sentence, in experience, the logical argumentative path of the decision is explicated only later, with respect to the decision itself. As a consequence, one of the apprehensions that leads to looking at the utilization of automatic systems like the ISS in the juridical field with diffidence, would crumble. In effect, in these conditions, the system does not impede the juridical operator's liberty of determination, nor does it influence the legal canons that he must respect; rather, it verifies, or at any rate, contributes to verifying, as it were, the excellence of the product of the judicial system [52,57].

References


[38] A.F. Dragoni, *Belief Revision: from Theory to Practice*, in *The Knowledge
Appendix A

Given a Knowledge Base KB, we calculate goods and nogoods by means of the algorithm described in [14] to calculate model-based diagnoses. In order to keep the paper self-contained, we recall here the basic definitions.
Let $F$ be a collection of sets. A hitting-set for $F$ is a set $H \subseteq \bigcup_{S \in F} S$ such that $H \cap S \neq \emptyset$ for each $S \in F$. A hitting-set for $F$ is minimal if none of its proper subsets is a hitting-set for $F$.

**Example.** Let $F = \{\{1,5,7\}, \{2,5,6\}, \{1,2\}\}$. The minimal hitting sets for $F$ are: $\{1,2\}$, $\{1,5\}$, $\{1,6\}$, $\{5,2\}$, $\{7,6,1\}$ and $\{7,2\}$. ■

An HS-Tree for $F$ is a smallest labeled tree $T$ such that:

1. its root is labeled with “√” if $F = \emptyset$; otherwise its root is labeled with a set of $F$
2. if $n$ is a node of $T$, define $H(n)$ be the set of the edge labels on the path from the node to the root. If $n$ is labeled with $\sqrt{}$, then it has no successor nodes in $T$. If $n$ is labeled with an element $\Sigma$ of $F$, then for each element $\sigma \in \Sigma$, $n$ has a successors node $n_{\sigma}$ joined with $n$ by an arc labeled with $\sigma$. The label for $n_{\sigma}$ is a set $S \in F$ such that $S \cap H(n_{\sigma}) = \emptyset$ if it exists, otherwise $n_{\sigma}$ is labeled with $\sqrt{}$

If $n$ is labeled with $\sqrt{}$, then $H(n)$ is a hitting-set for $F$. The set of all the $H(n)$ of the nodes labeled with $\sqrt{}$ includes all the minimal hitting-sets for $F$. An HS-Tree generated and pruned according to the following five rules, minimizes the accesses to $F$ while preserving all the paths, from the leaves to the root, corresponding to the minimal hitting sets.

1. The HS-Tree must be generated “breadth first” (each level $i$ must be completely generated in left-to-right order before generating the level $i+1$)
2. If a node $n$ is labeled with $S$ and $n'$ is another node such that $H(n') \cap S = \emptyset$, then label $n'$ with $S$
3. If a node $n$ is labeled with $\sqrt{}$ and $n'$ is another node such that $H(n) \subseteq H(n')$, then $n'$ must be pruned
4. If a node $n$ has already been generated and $n'$ is another node such that $H(n) = H(n')$, then $n'$ must be pruned
5. If a node $n$ is labeled with $S$ and $n'$ is labeled with $S'$ with $S' \subseteq S$, then for each $\sigma \in S - S'$, the arc starting from $n$ and labeled with $\sigma$ must be pruned

Given an HS-Tree for $F$ generated and pruned with these rules, the set:

$$\{H(n) \mid n \text{ is a node of } T \text{ labeled with } \sqrt{}\}$$

is the collection of all the minimal hitting sets for $F$. 

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Given KB, the collection of the goods and the collection of the nogoods are dual. If we remove from KB exactly one element for each nogood, what remains is a good. Hence the collection of the goods can be found by calculating all the minimal hitting-sets for the collection N of the nogoods, and keeping the complement of each of them w.r.t. KB.

If F is a collection of sets and F′ ⊆ F is the collection of all the minimal elements (w.r.t. set inclusion) of F, then F and F′ have the same minimal hitting-sets. This simplifies our task since we do not need to calculate the collection N of the nogoods (which are minimally inconsistent subsets of KB) but just a collection M ⊇ N of inconsistent subsets of KB, that is much easier. Given a collection F of sets, in any HS-Tree T relative to F, generated and pruned with the rules 1-5, every minimal element of F appears as the label of at least one node of T. So, if N ⊆ M, then all the nogoods (elements of N) will be used as labels of the nodes of T. During the generation of a node n, the rule 5 imposes to check if there is a label S which is a superset of the just calculated label S′ for n, so we have simply to eliminate such an S if it exists. Finding the nogoods is also useful for “Step 3” of the revision process since the belief function formalism revises the weights especially on the basis of minimal contradictions.

The collection M ⊇ N can be generated while generating the HS-Tree. After reducing in clausal normal form all the sentences in KB, we start a refutation process on it. If we find the empty clause, we label the root of T with the set of clauses in KB that have been involved in the refutation. Each clause σ_m in this node labels an arc toward a successor node. To calculate the label of this successor node we start a tentative refutation on KB \ σ_m. If KB \ σ_m is consistent, then the node is labelled with 1, otherwise it is labelled with the proper subset of KB \ σ_m from which the empty clause has been derived. The process iterates until there are no more successor nodes. Summarizing, the following algorithm generates all the goods and the nogoods in KB. Inc(C) is a function on set of clauses that returns 1 if C is consistent, returns a set D ⊆ C if D is inconsistent.

1. NG := ∅, G := ∅
2. When a node n needs a new label, give it the label Inc(KB \ H(n)) and add it to NG
3. When the rule 5 applies, eliminate S from NG
4. For each H(n) such that n is labelled with 1, put KB \ H(n) in G
5. Return NG and G